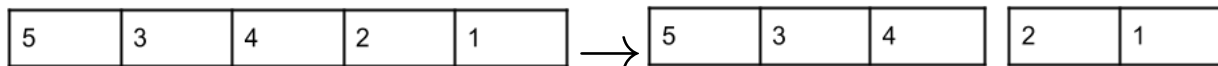


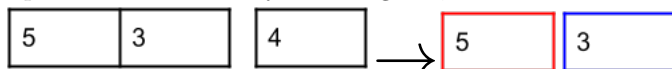
Mergesort

1 Mergesort

Mergesort is one of the most consistently fast algorithms. Mergesort works as follows: we divide the array in half, we perform mergesort on each half (recursion), we go through these steps until there is 1 element, then we move back up merging each sorted half. We will now perform a demo of Mergesort.



In this initial step, we start off with an unsorted array, we then start merge sort by splitting our array into 2 parts. Since our array is of length 5, we will make 1 side 1 element larger than the other.



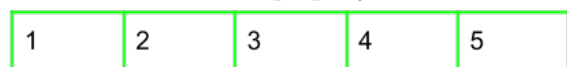
We will now recurse on the left array. We will split up the elements once again so that we have 2 arrays, one of the elements { 5, 3 } and 1 that is {4}. We will once again move the the left array and call mergesort on it. We note that when we call merge sort, each item has only 1 element, so both these elements are "sorted" in their own sub array.



In the first step listed, we "merge" 5 and 3. Before we continue, we must call mergesort on 4. We note that 4 is a sole element and that it is sorted. We will then merge both these arrays and we get the sorted array {3, 4, 5 }



We have 2 sub arrays left, one unsorted and one sorted. We call merge sort on the right array and sort those 2 elements so now it is properly ordered. We finally have 2 sub arrays that are sorted.



We merge our 2 arrays and finally have our sorted array.

The runtime of mergesort is $\Theta(N \log(N))$. We will utilize our knowledge of asymptotics to help us with this analysis. Each level does work proportional to the size of N (the size of the array/sub array). At each level, the array is split (the array is halved). In asymptotic notation, this looks like

$$\sum_{i=0}^{\log(n)} \frac{n}{2^i} \rightarrow \text{sum}_{i=0}^{\log(n)} n \rightarrow n \log n$$

The space complexity of merge sort is $O(N)$ because you would need to maintain a call stack for for your elements. However, once the call stack is used, you will be able to reuse it which prevents space complexity from being any worse.

Mergesort is a stable sort because anytime we run into 2 elements that are the same, we would just choose the element in the left array to be put in first.